Kinnelon High School
Mathematics Department
Summer Guide in Preparation of the 2019-2020 School Year

Dear students and families,

Please use the summer months to review and/or complete this Summer Guide regarding upcoming math courses for the 2019-2020 school year. All textbooks will be provided to students through the school in September. Mandatory course materials must be purchased. Suggested course materials are optional.

Sincerely,
The KHS Math Department

Course Name	Calculus CP
Textbook(s)	Calculus, Finney
Mandatory Course Materials	3 Ring Binder with loose leaf paper (suggested) or Notebook and Folder Calculator
Suggested Course Materials	If you are considering purchasing a graphing calculator, the TI-84 is the recommended calculator
Guide Summary	Please review the following topics prior to entering Calculus CP: - Unit circle: https://www.youtube.com/watch?v=hlxAfp2MeDM https://www.youtube.com/watch?v=cIVpemcoAlY - Basic functions: https://www.sandiegounified.org/schools/sites/default/files_link/schools/files/Domain/7755/TheTwelveBasicFunctions.pdf - Difference quotient: http://www.coolmath.com/precalculus-review-calculus-intro/precalculus-algebra/24-the-difference-quotient-01

 Name_____

Calculus CP Summer Assignment 2019

Evaluate each limit.

1) $\lim_{x\to 2}\left(x^3-4x^2+2\right)$

2) $\lim_{x\to -2}\left(2x+3\right)$

3) $\lim_{x\to 3}\dfrac{x-3}{x^2-4x+3}$

4) $\lim_{x\to -1}\dfrac{x^2+4x+3}{x+1}$

5) $\lim_{x\to 1}\dfrac{\sqrt{x}-1}{x-1}$

6) $\lim_{x\to 1}\dfrac{x-1}{\sqrt{x+8}-3}$

7) $\lim_{x\to -\infty}\dfrac{x-3}{x^2+x+1}$

8) $\lim_{x\to -\infty}-\dfrac{x}{x-3}$

9) $\lim_{x\to \infty}\dfrac{x^3}{2x^2+3}$

10) $\lim_{x\to \infty}\left(x^5-2x^3+x+2\right)$

Differentiate each function with respect to x.

11) $y = x$

12) $y = x^5 + 5x^3 + 4$

13) $y = 4x + x^{-1} + 2x^{-3}$

14) $y = -2x + 3x^{-2}$

15) $y = 3x^{\frac{5}{3}}$

16) $y = \sqrt[5]{x}$

17) $y = (-5x^5 + 3)(-2x^5 - 4)$

18) $y = (5x^4 + 4)(3x^4 - 2)$

19) $y = \dfrac{3}{x^3 + 2}$

20) $y = \dfrac{3x^5 + 5}{2x^2 + 3}$

For each problem, find the indicated derivative with respect to x.

21) $y = 3x^4$ Find $\frac{d^3y}{dx^3}$

22) $y = x^5 + 5x^3 + 2x^2$ Find $\frac{d^2y}{dx^2}$

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

23) $y = -2x^2 + 12x - 17$ at $(2, -1)$

24) $y = -x^3 + 3x^2 - 2$ at $(1, 0)$

For each problem, find the points where the tangent line to the function is horizontal. Indicate if no horizontal tangent line exists.

25) $y = -x^3 + 3x^2 - 1$

26) $y = -x^3 + 3x^2 - 2$

Differentiate each function with respect to x.

27) $y = (5x^4 - 3)^3$

28) $y = \sqrt[3]{-3x^2 + 1}$

29) $y = (3x^3 + 5)^3(-3x^2 + 1)$

30) $y = (2x + 3)^5(-5x^3 + 4)$

31) $y = \dfrac{x^4 + 3}{(4x + 3)^5}$

32) $y = \dfrac{(2x + 1)^3}{x^2 - 3}$

33) $y = \sec\left(4x^2\right)$

34) $y = \sin\left(2x^4\right)$

35) $y = \cos\left(5x^4\right)$

36) $y = \cot\left(5x^2\right)$

37) $y = \left(4x^3 + 3\right)\tan\left(4x^4\right)$

38) $y = \left(5x^3 + 2\right)\csc\left(3x^5\right)$

39) $y = \left(4x^3 + 1\right)\cos\left(4x^5\right)$

40) $y = e^{4x^3}$

41) $y = \ln 3x^3$

42) $y = e^{2x^5}$

43) $y = \ln x^2$

44) $y = \log_3 x^4$

45) $y = 2^{2x^4}$

46) $y = 3^{2x^2}$

47) $y = \log_2 2x^5$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

48) $5x = -5y^2 + 5y$

49) $x = 4y^2 + 3$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ at the given point.

50) $2x^3 = y^2 + 1$ at $(1, 1)$

51) $5y^3 = 4x + xy^2$ at $(-1, -1)$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$, the acceleration function $a(t)$, and the times t when the particle changes directions (when the particle is at rest).

52) $s(t) = t^3 - 18t^2 + 81t$

53) $s(t) = -t^3 + 15t^2$

Solve each optimization problem.

54) A rancher wants to construct two identical rectangular corrals using 100 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?

55) A supermarket employee wants to construct an open-top box from a 14 by 30 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?

56) A graphic designer is asked to create a movie poster with a 162 in^2 photo surrounded by a 4 in border at the top and bottom and a 2 in border on each side. What overall dimensions for the poster should the designer choose to use the least amount of paper?

57) Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 2048 ft^3 of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

Evaluate each indefinite integral. (What function has this function as its antiderivate?)

58) $\int -12x^3\,dx$

59) $\int (-2x+3)\,dx$

60) $\int (6x^5+3x^2)\,dx$

61) $\int (-24x^5+6x)\,dx$

62) $\int \left(6x + 6x^{-3}\right) dx$

63) $\int \left(25x^4 + 3x^{-4}\right) dx$

64) $\int -\frac{4}{x^2}\, dx$

65) $\int -\frac{3}{x^4}\, dx$

66) $\int 4\csc^2 x\, dx$

67) $\int -5\sin x\, dx$

68) $\int -2\sec x \tan x\, dx$

69) $\int -\sec^2 x\, dx$

70) $\int \cos x\, dx$

71) $\int -4\csc x \cot x\, dx$

72) $\int e^x\, dx$

73) $\int 4x^{-1}\, dx$

74) $\int \frac{3}{x}\, dx$

75) $\int -3e^x\, dx$

Answers to Calculus CP Summer Assignment 2019

1) -6

2) -1

3) $\frac{1}{2}$

4) 2

5) $\frac{1}{2}$

6) 6

7) 0

8) -1

9) ∞

10) ∞

11) $\frac{dy}{dx} = 1$

12) $\frac{dy}{dx} = 5x^4 + 15x^2$

13) $\frac{dy}{dx} = 4 - \frac{1}{x^2} - \frac{6}{x^4}$

14) $\frac{dy}{dx} = -2 - \frac{6}{x^3}$

15) $\frac{dy}{dx} = 5x^{\frac{2}{3}}$

16) $\frac{dy}{dx} = \frac{1}{5x^{\frac{4}{5}}}$

17) $\frac{dy}{dx} = (-5x^5 + 3) \cdot -10x^4 + (-2x^5 - 4) \cdot -25x^4$

$= 100x^9 + 70x^4$

18) $\frac{dy}{dx} = (5x^4 + 4) \cdot 12x^3 + (3x^4 - 2) \cdot 20x^3$

$= 120x^7 + 8x^3$

19) $\frac{dy}{dx} = -\frac{3 \cdot 3x^2}{(x^3 + 2)^2}$

20) $\frac{dy}{dx} = \frac{(2x^2 + 3) \cdot 15x^4 - (3x^5 + 5) \cdot 4x}{(2x^2 + 3)^2}$

21) $\frac{d^3y}{dx^3} = 72x$

22) $\frac{d^2y}{dx^2} = 20x^3 + 30x + 4$

23) $y = 4x - 9$

24) $y = 3x - 3$

25) $(0, -1), (2, 3)$

26) $(0, -2), (2, 2)$

27) $\frac{dy}{dx} = 3(5x^4 - 3)^2 \cdot 20x^3$

$= 60x^3(5x^4 - 3)^2$

28) $\frac{dy}{dx} = \frac{1}{3}(-3x^2 + 1)^{-\frac{2}{3}} \cdot -6x$

$= -2x(-3x^2 + 1)^{-\frac{2}{3}}$

29) $\frac{dy}{dx} = (3x^3 + 5)^3 \cdot -6x + (-3x^2 + 1) \cdot 3(3x^3 + 5)^2 \cdot 9x^2$

30) $\frac{dy}{dx} = (2x + 3)^5 \cdot -15x^2 + (-5x^3 + 4) \cdot 5(2x + 3)^4 \cdot 2$

31) $\frac{dy}{dx} = \frac{(4x + 3)^5 \cdot 4x^3 - 20(x^4 + 3)(4x + 3)^4}{(4x + 3)^{10}}$

32) $\frac{dy}{dx} = \frac{6(x^2 - 3)(2x + 1)^2 - (2x + 1)^3 \cdot 2x}{(x^2 - 3)^2}$

33) $\frac{dy}{dx} = \sec 4x^2 \tan 4x^2 \cdot 8x$

$= 8x \sec 4x^2 \tan 4x^2$

34) $\frac{dy}{dx} = \cos 2x^4 \cdot 8x^3$

$= 8x^3 \cos 2x^4$

35) $\frac{dy}{dx} = -\sin 5x^4 \cdot 20x^3$

$= -20x^3 \sin 5x^4$

36) $\frac{dy}{dx} = -\csc^2 5x^2 \cdot 10x$

$= -10x \csc^2 5x^2$

37) $\frac{dy}{dx} = (4x^3 + 3) \cdot \sec^2 4x^4 \cdot 16x^3 + \tan 4x^4 \cdot 12x^2$

38) $\frac{dy}{dx} = (5x^3 + 2) \cdot -1 \csc 3x^5 \cot 3x^5 \cdot 15x^4 + \csc 3x^5 \cdot 15x^2$

39) $\frac{dy}{dx} = (4x^3 + 1) \cdot -1 \sin 4x^5 \cdot 20x^4 + \cos 4x^5 \cdot 12x^2$

40) $\frac{dy}{dx} = e^{4x^3} \cdot 12x^2$

41) $\frac{dy}{dx} = \frac{1}{3x^3} \cdot 9x^2$

$= \frac{3}{x}$

42) $\frac{dy}{dx} = e^{2x^5} \cdot 10x^4$

43) $\frac{dy}{dx} = \frac{1}{x^2} \cdot 2x$
$= \frac{2}{x}$

44) $\frac{dy}{dx} = \frac{1}{x^4 \ln 3} \cdot 4x^3$
$= \frac{4}{x \ln 3}$

45) $\frac{dy}{dx} = 2^{2x^4} \ln 2 \cdot 8x^3$
$= x^3 \cdot 2^{2x^4+3} \ln 2$

46) $\frac{dy}{dx} = 3^{2x^2} \ln 3 \cdot 4x$

47) $\frac{dy}{dx} = \frac{1}{2x^5 \ln 2} \cdot 10x^4$
$= \frac{5}{x \ln 2}$

48) $\frac{dy}{dx} = \frac{1}{-2y+1}$

49) $\frac{dy}{dx} = \frac{1}{8y}$

50) $\left.\frac{dy}{dx}\right|_{\substack{x=1 \\ y=1}} = 3$

51) $\left.\frac{dy}{dx}\right|_{\substack{x=-1 \\ y=-1}} = \frac{5}{13}$

52) $v(t) = 3t^2 - 36t + 81$, $a(t) = 6t - 36$
Changes direction at: $t = \{3, 9\}$

53) $v(t) = -3t^2 + 30t$, $a(t) = -6t + 30$
Changes direction at: $t = \{10\}$

54) A = the total area of the two corrals x = the length of the non-adjacent sides of each corral
Function to maximize: $A = 2x \cdot \frac{100 - 4x}{3}$ where $0 < x < 25$
Dimensions of each corall: $\frac{25}{2}$ ft (non-adjacent sides) by $\frac{50}{3}$ ft (adjacent sides)

55) V = the volume of the box x = the length of the sides of the squares
Function to maximize: $V = (30 - 2x)(14 - 2x) \cdot x$ where $0 < x < 7$
Sides of the squares: 3 in

56) A = the area of the poster x = the width of the photo
Function to minimize: $A = (x + 2 \cdot 2)\left(\frac{162}{x} + 2 \cdot 4\right)$ where $0 < x < \infty$
Dimensions of the entire poster: 13 in wide by 26 in tall

57) A = the area of the glass x = the length of the sides of the square bottom
Function to minimize: $A = x^2 + 4x \cdot \frac{2048}{x^2}$ where $0 < x < \infty$
Dimensions of the aquarium: 16 ft by 16 ft by 8 ft tall

58) $-3x^4 + C$

59) $-x^2 + 3x + C$

60) $x^6 + x^3 + C$

61) $-4x^6 + 3x^2 + C$

62) $3x^2 - \frac{3}{x^2} + C$

63) $5x^5 - \frac{1}{x^3} + C$

64) $\frac{4}{x} + C$

65) $\frac{1}{x^3} + C$

66) $-4\cot x + C$

67) $5\cos x + C$

68) $-2\sec x + C$

69) $-\tan x + C$

70) $\sin x + C$

71) $4\csc x + C$

72) $e^x + C$

73) $4\ln|x| + C$

74) $3\ln|x| + C$

75) $-3e^x + C$